

MODIFIED BOX-JENKINS AND GARCH FOR  
FORECASTING HIGHLY VOLATILE TIME  
SERIES DATA

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DOCTOR OF PHILOSOPHY

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## **SUPERVISOR'S DECLARATION**

I hereby declare that I have checked this thesis and in my opinion, this thesis is adequate in terms of scope and quality for the award of the degree of Doctor of Philosophy.

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## **STUDENT'S DECLARATION**

I hereby declare that the work in this thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at Universiti Malaysia Pahang or any other institutions.

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## ABSTRAK

Model Box-Jenkins digunakan secara meluas sama ada sebagai model peramalan, piawaian atau bersepadu dalam kajian terkini siri masa. Pemodelan Box-Jenkins adalah salah satu teknik peramalan paling berkuasa yang digunakan dalam praktis kajian analisis siri masa. Kebanyakan data siri masa contohnya data ekonomi dan sains persekitaran adalah bervarians tidak malar secara semulajadi. Walau bagaimanapun, untuk data siri masa yang bervarians tidak malar yang tinggi, model Box-Jenkins adalah tidak sesuai untuk diaplikasikan kerana andaian ralat varians malar tidak dipenuhi dan ia juga tidak dapat mengendalikan sifat heteroskedastisiti. Menggabungkan model Box-Jenkins dengan model stokastik heteroskedastisiti seperti model *generalised autoregressive conditional heteroscedastic* (GARCH) merupakan satu kaedah yang berkesan untuk mengatasi kekangan model Box-Jenkins bagi data varians tidak malar. Kajian ini menilai prestasi model kombinasi antara Box-Jenkins dan variasi GARCH dalam pemodelan dan peramalan data univariat siri masa yang bervarians tidak malar yang tinggi dengan pemodelan Box-Jenkins sebagai asas prosedur. Empat prosedur dicadangkan dalam kajian ini dalam menilai prestasi model kombinasi tersebut di mana tiga cadangan prosedur awal adalah menggunakan model Box-Jenkins dengan standard GARCH (or BJ-G). Prosedur cadangan pertama adalah berdasarkan prosedur asas Box-Jenkins dan ia digunakan sebagai kajian tinjauan awal. Prosedur cadangan kedua adalah berdasarkan prosedur cadangan pertama yang difokuskan untuk mengendalikan data siri masa bervarians tidak malar yang tinggi secara spesifik, menggunakan model BJ-G dengan penekanan kepada pengecaman ciri data bervarians tidak malar yang tinggi pada peringkat awal. Manakala, prosedur cadangan ketiga adalah lanjutan daripada prosedur cadangan kedua, yang digunakan untuk menilai keupayaan model BJ-G untuk peramalan jangka panjang. Prosedur cadangan keempat adalah kombinasi prosedur cadangan kedua dan ketiga yang mana ia merupakan prosedur komprehensif untuk pemodelan dan peramalan data siri masa yang bervarians tidak malar yang tinggi menggunakan model Box-Jenkins – variasi GARCH. Kesemua prosedur cadangan diilustrasikan dengan data harian harga emas dunia kerana data ini adalah data siri masa yang bervarians tidak malar yang tinggi. Berdasarkan kajian awal ke atas 5000 data harian data harian harga emas menggunakan prosedur cadangan pertama BJ-G, nilai ralat yang kecil membuktikan model BJ-G adalah model yang diyakini untuk pemodelan dan peramalan data bervarians tidak malar yang tinggi. Keputusan empirik daripada data harian harga emas dunia menggunakan prosedur cadangan kedua menyatakan prosedur ini adalah lebih praktikal berbanding prosedur cadangan pertama dalam pemodelan data bervarians tidak malar yang tinggi menggunakan model BJ-G dan secara langsung dapat menentukan bilangan data yang optimal. Keputusan empirik mencadangkan 25% daripada data yang terkini atau 1250 data adalah mencukupi untuk model BJ-G dengan prestasi peramalan yang sama seperti menggunakan kesemua data. Manakala, berdasarkan kajian empirik ke atas 1250 data harian harga emas itu menggunakan prosedur cadangan ketiga, didapati model BJ-G berkeupayaan untuk mengikuti pola data sebenar sehingga tujuh hari ke hadapan, khasnya dalam selang peramalan 95%. Prosedur cadangan keempat diuji ke atas model Box-Jenkins dengan variasi GARCH menggunakan data siri yang sama digunakan untuk prosedur cadangan ketiga. Sebagai kesimpulan, model kombinasi Box-Jenkins dan variasi GARCH mempunyai potensi yang besar, oleh itu prosedur cadangan keempat BJ-G memberikan satu prosedur peramalan siri masa yang komprehensif, sistematik dan praktikal bagi data siri masa yang bervarians tidak malar yang tinggi.

## ABSTRACT

The Box-Jenkins model has widely been used either as the forecasting, benchmarking or as the integrated model in the current research of time series. The Box-Jenkins modelling is one of the most powerful forecasting techniques available in research practice of the time series analysis. Most of the time series data such as in economics and in environmental sciences are volatile in nature. However, for a highly volatile time series data, the Box-Jenkins model is inappropriate to be applied since it violates the errors assumption of constant variance and it is not able to handle the heteroscedasticity property. Combining the model with a heteroscedastic stochastic model such as the generalised autoregressive conditional heteroscedastic model (GARCH) can be an effective way to overcome the limitation of the Box-Jenkins model in handling the non-constant variance. This study evaluates the performance of the combination model of Box-Jenkins and GARCH-type in modelling and forecasting univariate highly volatile time series data with Box-Jenkins modelling as the base procedure. In evaluating the performance of the model, four procedures are proposed in this study where the first three procedures are using the model of Box-Jenkins and standard GARCH (or BJ-G). The first proposed procedure is developed based on the theoretical Box-Jenkins's procedure and it is used for the preliminary study. The second proposed procedure is developed based on the first proposed procedure to focus on handling the highly volatile time series data specifically, using BJ-G model by emphasizing on the identification of highly volatile characteristic in the data at the early stage. While the third proposed procedure is an extension from the second procedure, which evaluates the multistep ahead forecasting performance of the BJ-G model. The fourth procedure of BJ-G is developed from the second and third procedures and it is a comprehensive procedure for modelling and forecasting highly volatile time series data using Box-Jenkins – GARCH-type model. The proposed procedures are illustrated using the daily world gold price data since it is a highly volatile type of time series. Based on the preliminary study on 5000 world daily gold price data set using the first procedure of BJ-G, the small magnitude of error proves that BJ-G is a reliable model in modelling and forecasting highly volatile data. The empirical results of the world daily gold price using the second proposed procedure indicate that the procedure is more practical than the first propose procedure to be used in modelling a univariate highly volatile data using BJ-G model which simultaneously ensures an optimal number of data in dealing with the model. The empirical results suggested that the latest 25% of data or 1250 data is sufficient to be employed using BJ-G model with similar forecasting performance as by using all data. Meanwhile, based on the empirical results on the 1250 world daily gold prices and by employing the third procedure, it is found that the BJ-G model is able to follow the trend of the actual data up to seven days ahead, specifically within 95% prediction interval. The fourth proposed procedure is also tested on the Box-Jenkins with various GARCH-type models using the same data series as in the third proposed procedure. In conclusion, the combination model of Box-Jenkins and GARCH-type has great potential, thus the fourth proposed procedure of BJ-G provides a comprehensive, systematic and practical procedure of time series forecasting for univariate highly volatile time series data.

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## LIST OF SYMBOLS

$y_t$	The original series or the observed data at time $t$ period
$y_t^*$	The transformed data at time $t$
$s_t$	The stationary series at time $t$
$h$	The forecasting horizon
$\hat{y}_{T+h}$	The forecast data for $h$ -step ahead
$\hat{s}_{T+h}$	The simulated stationary series for forecasting horizon $h$
$T$	The number of data in-sample series; the origin
$n$	The number of data in out-of-sample series
$k$	The number of lag
$k_{\max}$	The maximum number for lag $k$
$w$	The number of series
$m$	The number of auxiliary regressors
$\mu$	The mean data (or model)
$\mu_t$	The conditional mean of $s_t$
$\sigma^2$	The variance data (or model)
$\sigma_X^2$	The variance of $X$
$\sigma_t^2$	The conditional variance of $y_t$
$\sigma_t$	The volatility of $a_t$
$a_t$	The random errors at time $t$ period
$\{\hat{a}_t\}$	The residuals of data
$\hat{a}_t$	The residual at time $t$
$v_t$	The error in the squared returns, that is $a_t^2 - \sigma_t^2$
$\varepsilon_t$	The standardised error (innovations) of model
$e_T(h)$	The $h$ -step ahead forecast error at origin $T$
$\gamma_k$	Autocovariance coefficient at lag $k$
$\{\gamma_k\}$	The plot of $\gamma_k$ versus lag $k$ or the autocovariance function
$\rho_k$	Autocorrelation coefficient at lag $k$

$\{\rho_k\}$	The plot of $\rho_k$ versus lag $k$ or the autocorrelation function
$r_k$	The sample $\rho_k$
$\phi_{kk}$	Partial autocorrelation coefficient at lag $k$
$r_{kk}$	The sample $\phi_{kk}$
$p$	The order of the autoregressive model
$q$	The order of the moving average model
$\varphi_p$	The autoregressive parameters with order $p$
$\theta_q$	The moving average parameters with order $q$
$d$	The order of differencing
$P$	The order of seasonal autoregressive
$Q$	The order of the seasonal moving average
$D$	The order of seasonal differencing
$S$	The seasonal period
$B$	The backward shift operator
$r$	The order of the generalised autoregressive conditional heteroskedastic model
$s$	The order of the autoregressive conditional heteroskedastic model
$\alpha_i$	The coefficient of the parameters ARCH
$\beta_i$	The coefficient of the parameters GARCH
$c$	The constant
$C_i$	The coefficient for the $\Delta y_{t-i}$
$X$	The continuous random variable
$f(x)$	The probability density function of $X$
$K(x)$	The kurtosis of $X$
$S(x)$	The skewness of $X$
$m'_\ell$	The $\ell$ th moment of a continuous random variable $X$ about the origin
$m_\ell$	The $\ell$ th central moment of $X$ about the mean
$\lambda$	The minimum residual mean square error value
$\hat{\lambda}$	The estimated $\lambda$
$J(\lambda; y)$	The Jacobian of the transformation

$S(\lambda)$	The residual sum of squares in the analysis of variance of $y_t^*$
$\nu_\lambda$	The number of independent components in $\lambda$
$\nu$	The degrees of freedom
$\xi$	The skewness parameter
$\kappa$	The shape parameter
$\Gamma(\cdot)$	The gamma function
$a$	The $(T \times T)$ matrix
$\theta$	The $(T \times 1)$ vector of unknown parameters associated with the transformed data
$\Gamma_T$	The covariance matrix of symmetric form
$P_T$	The autocorrelation matrix
$L$	The likelihood function
$\ln L$	The log likelihood function
$\ln L_{\max}$	The maximised log likelihood function
$Z$	The integers
$x'_t$	The deterministic time trend
$\phi_a$	The parameter that defines the relationship between successive values of $a_t$ and $a_{t-1}$
$\phi_{df}$	The parameter to be estimated in DF-test and ADF-test
$\pi$	The parameter to be estimated in DF-test and ADF-test where $\pi = \phi - 1$
$\hat{b}$	The estimated coefficient for ARCH-LM
$TD_t$	The deterministic terms
$M$	The risk premium parameter
$N_{t-i}$	The indicator for negative $a_{t-i}$
$g_i$	The leverage effect term of $a_{t-i}$
$\delta$	The positive real number

## LIST OF ABBREVIATIONS

AR	Autoregressive model
MA	Moving average model
ARMA	Autoregressive moving average model
ARIMA	Autoregressive integrated moving average model
SARIMA	Seasonal autoregressive integrated moving average model
ACF	Autocorrelation function
PACF	Partial autocorrelation function
MLE	Maximum likelihood estimation
OLS	Ordinary least squares
ARCH	Autoregressive conditional heteroskedastic
GARCH	Generalised autoregressive conditional heteroskedastic
ARCH LM	ARCH Lagrange Multiplier
EACF	Extended autocorrelation function
IID	Independent identically distributed
NID	Normal independently distributed
pdf	Probability density function
AIC	Akaike Information Criteria
SIC	Schwarz Information Criterion
ADF	Augmented Dickey-Fuller
SSR	Residual sum of squares
dof	Degrees of freedom
MAE	Mean absolute error
MSE	Mean square error
RMSE	Root mean square error
MAPE	Mean absolute percentage error
PIs	Prediction intervals
CV	Cross-validation
LBQ-test	Ljung-Box $Q$ -test
DW-test	Durbin-Watson test
JB-test	Jarque-Bera test

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